

# Wavelets on nuclear interactions and QCD on jet multiplicities

I.M. Dremin

*Lebedev Physical Institute, Moscow 117924, Russia*

## Abstract

This talk at Fradkin conference is devoted to brief description of latest results in two topics I worked on during last years.

The multiresolution analysis and fast wavelet transform became a standard procedure for pattern recognition and I describe how it has been used for analysis of high energy nucleus-nucleus interactions<sup>1</sup>.

Latest QCD results on multiplicities in quark and gluon jets are discussed and confronted to experimental data<sup>2</sup>.

## 1 Introduction

This talk is given at Fradkin memorial conference on Friday, 9 June 2000. Friday was always a traditional day of seminars on quantum field theory in our department, so-called Fradkin seminars. Two talks on wavelets were given at this seminar last year just before Fradkin was put in a hospital. Therefore I decided to talk here about wavelet application in particle physics and gave to organizers of the conference the title containing the first part of the above one. However soon I learned that the topic on QCD (the second part of the present title) would fit the conference schedule better and asked organizers for replacement. It was shifted to QCD session but with an old title on wavelets. Thus I decided to give two talks at one, and it explains how these two topics appear here together.

## 2 Wavelets

First I learned about wavelets from Pete Carruthers in 1993. He applied them [1, 2, 3] for analysis of some scaling cascade models used, in particular, in multiparticle production modelling. It was briefly described in our review paper [4]. Then I proposed to use wavelets for pattern recognition in high

---

<sup>1</sup>Other applications can be found, e.g., in Web site [www.awavelet.com](http://www.awavelet.com).

<sup>2</sup>Detailed review of this subject is given in [15]

energy nucleus-nucleus collisions, and it was applied to experimental data [6].

Wavelets became a powerful mathematical tool in many investigations. They are used in those cases when the result of the analysis of a particular signal<sup>3</sup> should contain not only the list of its typical frequencies (scales) but also the knowledge of the definite local coordinates where these properties are important. Wavelets form a complete orthonormalized system of functions with a finite support by using dilations and translations. That is why by changing a scale (dilations) they can describe the local characteristics of a signal, and by translations they cover the whole region in which it is studied. Due to the completeness of the system, they also allow for the inverse transformation to be done. The locality property of wavelets leads to their substantial superiority over Fourier transform which provides us only with the knowledge of global frequencies (scales) of the object under investigation because the system of functions used (sine, cosine) is defined on the infinite interval.

High energy collisions of elementary particles result in production of many new particles in a single event. Each newly created particle is depicted kinematically by its momentum vector i.e. by a dot in the three-dimensional phase space. Different patterns formed by these dots in the phase space would correspond to different dynamics. To understand this dynamics is a main goal of all studies done at accelerators and in cosmic rays. Especially intriguing is a problem of the quark-gluon plasma, the state of matter with deconfined quarks and gluons which could exist during an extremely short intervals of time. One hopes to create it in collisions of high energy nuclei. Nowadays, the data about Pb-Pb collisions are available where in a single event more than 1000 particles are produced. We are waiting for RHIC accelerator in Brookhaven and LHC in CERN to provide events with up to 20000 new particles created. Therefore the problem of phase space pattern recognition in an event-by-event analysis becomes meaningful. It is believed that the detailed characterization of each collision event could reveal the rare new phenomena, and it will be statistically reliable due to a large number of particles produced in a single event.

When individual events are imaged visually, the human eye has a ten-

---

<sup>3</sup>The notion of a signal is used here for any recorded information about some processes, objects, functions etc.

dency to observe different kinds of intricate patterns with dense clusters (spikes) and rarefied voids. However, the observed effects are often dominated by statistical fluctuations. The method of factorial moments was proposed [5] to remove them but it is hard to use in event-by-event approach. The wavelet analysis avoids smooth polynomial trends typical for the statistical component. It was first applied [6] to analyze the individual high multiplicity event of Pb-Pb interaction at energy 158 GeV per nucleon. With emulsion technique used in experiment the angles of particle emission are often measured only, and the two-dimensional phase space is considered therefore. The experimental statistics is rather low but acceptance is high and homogeneous that is important for proper pattern recognition. To simplify the analysis, the two-dimensional target diagram representing the polar and azimuthal angles of created charged particles was split into 24 one-dimensional functions representing the polar angle distribution of these particles in 24 azimuthal angle sectors of  $\pi/12$  and in each of them particles were projected onto the polar angle  $\theta$  axis. Thus one-dimensional functions of the rapidity distribution of these particles in 24 sectors were obtained. Then the wavelet coefficients were calculated in all of them and tied up together (continuous MHAT wavelet was used). The resulting pattern showed that many particles are concentrated close to some value of the polar angle i.e. reveal the ring-like structure in the target diagram. The interest to such patterns is related to the fact that they can result from the so-called gluon Cherenkov radiation [7, 8] or, more generally, from the gluon bremsstrahlung at a finite length within a quark-gluon medium (plasma, in particular). More elaborated two-dimensional analysis was done recently [9] and confirmed these conclusions with jet regions tending to lie on some ring-like formations. The jet-like substructure of the event becomes more pronounced, and ring-like correlations of jetty regions are noticeable. With higher statistics, one can learn if the angular distribution of these rings corresponds to theoretical expectations. It is due to wavelet analysis that for the first time the fluctuation structure of an event is shown in a way similar to the target diagram on the two-dimensional plot.

Previously, some attempts [10, 11, 12] to consider such events with different methods of treating the traditional projection and correlation measures revealed just that such substructures lead to spikes in the angular (pseudorapidity) distribution and are somewhat jetty. Various Monte Carlo simulations of the process were compared to the data and failed to describe this jettiness

in its full strength. More careful analysis [13, 14] of large statistics data on hadron-hadron interactions (unfortunately, however, for rather low multiplicity) with dense groups of particles separated showed some "anomaly" in the angular distribution of these groups awaited from the theoretical side. Further analysis using the results of wavelet transform are needed to check this conclusion in high multiplicity nucleus-nucleus interactions when many events of this kind become available.

### 3 QCD

Here I briefly describe recent advances in theoretical understanding of multiplicity distributions of quark and gluon jets. The extended survey with more detailed comparison to experimental data can be found in the recent review paper [15].

The progress in experimental studies of properties of quark and gluon jets is very impressive. Therefore the study of the energy evolution of such parameters of multiplicity distributions of jets as their average multiplicities and widths becomes possible. It is well known that the average multiplicities of quark and gluon jets increase quite fast with energy but their ratio has a much slower dependence.

The perturbative QCD provides quite definite predictions which can be confronted to experiment. In brief, the results can be summarized by saying that the energy dependence of the mean jet multiplicity can be perfectly fitted but the ratio of gluon to quark jet multiplicities can be described within the precision of 15-20% only. Moreover, one can understand why next-to-leading approximation is good enough for describing the energy dependence, but it is not quite satisfactory yet for the ratio value. I show this by presenting the analytical expressions. For the corresponding Figures, I refer the reader to the review paper [15].

The theoretical asymptotical value of the ratio of average multiplicities equal 2.25 is much higher than its experimental values, which are in the range from 1.05 at comparatively low energies of  $\Upsilon$  resonance to 1.5 at  $Z^0$  resonance. The next-to leading order (NLO) corrections reduce this ratio from its asymptotical value by about 10% at  $Z^0$  energy. The NNLO and 3NLO terms diminish it further and show the tendency to approximate the data with better accuracy. The computer solution of QCD equations for

the generating functions has shown even better agreement with experiment not only on this ratio but on higher moments of multiplicity distributions as well. Being perfect at  $Z^0$  energy, the agreement in the ratio is not as good at lower energies where the theoretical curve is still about 15-20% above the experimental one. In other words, the theoretically predicted *slope of the ratio* of multiplicities in gluon and quark jets is noticeably smaller than its experimental value. Nevertheless, one can speak about the steady convergence of theory and experiment with subsequent improvements being done. Moreover, it is even surprising that any agreement is achieved in view of the expansion parameter being extremely large (about 0.5) at present energies.

The importance of studying the slopes stems from the fact that some of them are extremely sensitive (while others are not) to higher order perturbative corrections and to non-perturbative terms in the available energy region. Thus they provide us with a good chance to learn more about the structure of the perturbation series from experiment.

In the perturbative QCD, the general approach to studying the multiplicity distributions is formulated in the framework of equations for generating functions. Therefrom, one can get equations for average multiplicities and, in general, for any moment of the multiplicity distributions [16]. In particular, two equations for average multiplicities of gluon and quark jets are written as

$$\begin{aligned} \langle n_G(y) \rangle' = & \int dx \gamma_0^2 [K_G^G(x) (\langle n_G(y + \ln x) \rangle + \langle n_G(y + \ln(1-x)) \rangle - \langle n_G(y) \rangle) \\ & + n_f K_G^F(x) (\langle n_F(y + \ln x) \rangle + \langle n_F(y + \ln(1-x)) \rangle - \langle n_G(y) \rangle)], \end{aligned} \quad (1)$$

$$\langle n_F(y) \rangle' = \int dx \gamma_0^2 K_F^G(x) (\langle n_G(y + \ln x) \rangle + \langle n_F(y + \ln(1-x)) \rangle - \langle n_F(y) \rangle). \quad (2)$$

Herefrom one can learn about the energy evolution of the ratio of multiplicities  $r$  and of the QCD anomalous dimension  $\gamma$  (the slope of the logarithm of average multiplicity in a gluon jet) defined as

$$r = \frac{\langle n_G \rangle}{\langle n_F \rangle}, \quad \gamma = \frac{\langle n_G \rangle'}{\langle n_G \rangle} = (\ln \langle n_G \rangle)'. \quad (3)$$

Here, prime denotes the derivative over the evolution parameter  $y = \ln(p\Theta/Q_0)$ ,  $p$ ,  $\Theta$  are the momentum and the initial angular spread of the jet, related to the parton virtuality  $Q = p\Theta/2$ ,  $Q_0 = \text{const}$ ,  $K$ 's are the well known splitting functions,  $\langle n_G \rangle$  and  $\langle n_F \rangle$  are the average multiplicities in gluon and

quark jets,  $\langle n_G \rangle'$  is the slope of  $\langle n_G \rangle$ ,  $n_f$  is the number of active flavours. The perturbative expansion of  $\gamma$  and  $r$  is written as

$$\gamma = \gamma_0(1 - a_1\gamma_0 - a_2\gamma_0^2 - a_3\gamma_0^3) + O(\gamma_0^5), \quad (4)$$

$$r = r_0(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3) + O(\gamma_0^4), \quad (5)$$

where  $\gamma_0 = \sqrt{2N_c\alpha_S/\pi}$ ,  $\alpha_S$  is the strong coupling constant,

$$\alpha_S = \frac{2\pi}{\beta_0 y} \left[ 1 - \frac{\beta_1 \ln(2y)}{\beta_0^2 y} \right] + O(y^{-3}), \quad (6)$$

$\beta_0 = (11N_c - 2n_f)/3$ ,  $\beta_1 = (51N_c - 19n_f)/3$ ,  $r_0 = N_c/C_F$ , and in QCD  $N_c = 3$  is the number of colours,  $C_F = 4/3$ .

The limits of integration in eqs. (1), (2) used to be chosen equal either to 0 and 1 or to  $e^{-y}$  and  $1 - e^{-y}$ . This difference, being negligibly small at high energies  $y$ , is quite important at low energies. Moreover, it is of physics significance. With limits equal to  $e^{-y}$  and  $1 - e^{-y}$ , the partonic cascade terminates at the perturbative level  $Q_0$  as is seen from the arguments of multiplicities in the integrals. With limits equal to 0 and 1, one extends the cascade into the non-perturbative region with low virtualities  $Q_1 \approx xp\Theta/2$  and  $Q_2 \approx (1-x)p\Theta/2$  less than  $Q_0/2$ . This region contributes terms of the order of  $e^{-y}$ , power-suppressed in energy. It is not clear whether the equations and LPHD hypothesis are valid down to some  $Q_0$  only or the non-perturbative region can be included as well.

Nevertheless, the purely perturbative expansion (4), (5) with constant coefficients  $a_i, r_i$  and energy-dependent  $\gamma_0$  is at work just in the case of limits 0 and 1. The values of  $a_i, r_i$  for different number of active flavors  $n_f$  are tabulated in [17]. At  $Z^0$ -energy the subsequent terms in (5) diminish the value of  $r$  compared with its asymptotics  $r_0 = 2.25$  approximately by 10%, 13%, 1% for  $n_f = 4$  getting closer to experiment. However the theoretical value of  $r$  still exceeds its experimental values by 15-20%.

The energy dependence of mean multiplicities can be obtained [18, 17] from the definition (3) by inserting there the value of  $\gamma$  (4) and integrating over  $y$ . Keeping the terms as small as  $y^{-1}$  at large  $y$  in the exponent, one gets [17] the following expressions for energy dependence of multiplicities of gluon (G) and quark (F) jets

$$\langle n_G \rangle = Ky^{-a_1 C^2} \exp[2C\sqrt{y} + \delta_G(y)], \quad (7)$$

with  $K$  an overall normalization constant,  $C = \sqrt{4N_c/\beta_0}$ , and

$$\delta_G(y) = \frac{C}{\sqrt{y}} \left[ 2a_2 C^2 + \frac{\beta_1}{\beta_0^2} [\ln(2y) + 2] \right] + \frac{C^2}{y} \left[ a_3 C^2 - \frac{a_1 \beta_1}{\beta_0^2} [\ln(2y) + 1] \right]; \quad (8)$$

$$\langle n_F \rangle = \frac{K}{r_0} y^{-a_1 C^2} \exp [2C \sqrt{y} + \delta_F(y)], \quad (9)$$

with

$$\delta_F(y) = \delta_G(y) + \frac{C}{\sqrt{y}} r_1 + \frac{C^2}{y} (r_2 + \frac{r_1^2}{2}). \quad (10)$$

It happens that 2NLO and 3NLO terms (contributing to  $y^{-1/2}$  and  $y^{-1}$  terms in the exponent) are almost constant at present energies and do not change the energy dependence prescribed in NLO approximation. It explains why the energy dependence is well fitted by both NLO and 3NLO formulas while 2NLO correction to the ratio  $r$  is large and important.

The rather small difference in  $r$  values results in quite noticeable disagreement of the slopes  $r'$ . Theoretical estimates can be shown [17] to be quite predictive for the ratio of the slopes of multiplicities but it is much less reliable to use the perturbative estimates even at  $Z^0$ -energy for such quantities as the slope of  $r$  or the ratio of slopes of logarithms of multiplicities (the logarithmic slopes). Much higher energies are needed to do that. Thus the values of  $r'$  and/or of the logarithmic slopes can be used to verify the structure of the perturbative expansion.

I demonstrate it here on the example of the slope value. The slope  $r'$  is extremely sensitive to higher order perturbative corrections. The role of higher order corrections is increased here compared with  $r$  because each  $n$ th order term proportional to  $\gamma_0^n$  gets an additional factor  $n$  in front of it when differentiated, the main constant term disappears and the large ratio  $r_2/r_1$  becomes crucial:

$$r' = Br_0 r_1 \gamma_0^3 \left[ 1 + \frac{2r_2 \gamma_0}{r_1} + \left( \frac{3r_3}{r_1} + B_1 \right) \gamma_0^2 + O(\gamma_0^3) \right], \quad (11)$$

where the relation  $\gamma_0' \approx -B\gamma_0^3(1+B_1\gamma_0^2)$  has been used with  $B = \beta_0/8N_c$ ;  $B_1 = \beta_1/4N_c\beta_0$ . The factor in front of the bracket is very small already at present energies:  $Br_0 r_1 \approx 0.156$  and  $\gamma_0 \approx 0.5$ . However, the numerical estimate of  $r'$  is still indefinite due to the expression inside the brackets. Let us note that

each differentiation leads to a factor  $\alpha_S$  or  $\gamma_0^2$ , i.e., to terms of higher order. For values of  $r_1, r_2, r_3$  tabulated above ( $n_f = 4$ ) one estimates  $2r_2/r_1 \approx 4.9$ ,  $(3r_3/r_1) + B_1 \approx 1.5$ ). The simplest correction proportional to  $\gamma_0$  is more than twice larger 1 at energies studied and the next one is about 0.4. Therefore the ever higher order terms should be calculated for the perturbative values of  $r'$  to be trusted. The slope  $r'$  is equal to 0 for a fixed coupling constant.

The higher order terms are also important for the moments of the multiplicity distributions [19]. The normalized second factorial moment  $F_2$  defines the width of the multiplicity distribution.

The asymptotical ( $\gamma_0 \rightarrow 0$ ) values of  $F_2^G$  and  $F_2^F$  are different:

$$F_{2,as}^G = \frac{4}{3}, \quad F_{2,as}^F = \frac{7}{4}. \quad (12)$$

At  $Z^0$  energy the widths of the distributions are smaller

$$F_2^G \approx 1.12, \quad F_2^F \approx 1.34. \quad (13)$$

but still much larger than their experimental values 1.02 and 1.08, correspondingly. The rather large difference of the perturbative (13) and experimental values at  $Z^0$  indicates that moments of the distributions should be sensitive to corrections. The conclusions about the third moments are similar. Nonetheless, the computer solution of QCD equation happened to be quite successful in fitting experimental data even for higher moments and their ratios  $H_q$  introduced in [20]. It shows that the role of conservation laws treated approximately in the analytical approach and accurately accounted in computer calculations becomes more important for higher moments.

Thus it is shown that the analytical approach is quite successful in demonstrating that all features of QCD predictions about multiplicities of quark and gluon jets correspond to the general trends of experimental data. Some disagreement at the level of 15-20% is understandable due to incomplete account for the energy-momentum conservation in such an approach. Further accurate computer solutions are needed to check if these trends persist at the higher precision level.

## References

- [1] P. Carruthers, in Proc. "Hot and dense nuclear matter", Bodrum, 1993, p.65.



- [2] P. Lipa, M. Greiner, P. Carruthers, in Proc. "Soft physics and fluctuations", Krakow, 1993, p.105.
- [3] M. Greiner, J. Gieseemann, P. Lipa, P. Carruthers, Z. Phys. C69 (1996) 309.
- [4] E. DeWolf, I.M. Dremin, W. Kittel, Phys. Rep. 270 (1996) 1.
- [5] A. Bialas, R. Peschanski, Nucl. Phys. B273 (1988) 703.
- [6] N.M. Astafyeva, I.M. Dremin, K.A. Kotelnikov, Mod. Phys. Lett. A12 (1997) 1185.
- [7] I.M. Dremin, JETP Lett. 30 (1979) 140.
- [8] I.M. Dremin, Yad. Fiz. 33 (1981) 1357.
- [9] I.M. Dremin, O.V. Ivanov, S.A. Kalinin et al, Phys. Lett. B (to be published)
- [10] A.V. Apanasenko, N.A. Dobrotin, I.M. Dremin et al, JETP Lett. 30 (1979) 145.
- [11] EMU01 Collaboration, M.I. Adamovich et al, J. Phys. G19 (1993) 2035.
- [12] KLM Collaboration, M.L. Cherry et al, Acta Physica Polonica B29 (1998) 2129.
- [13] I.M. Dremin, P.L. Lasaeva, A.A. Loktionov et al, Sov. J. Nucl. Phys. 52 (1990) 535; Mod. Phys. Lett. A5 (1990) 1743.
- [14] N.M. Agababian et al, Phys. Lett. B389 (1996) 397.
- [15] I.M. Dremin, J.W. Gary, Phys. Rep. (to be published); hep-ph 0004215.
- [16] I.M. Dremin, V.A. Nechitailo, Mod. Phys. Lett. A9 (1994) 1471; JETP Lett. 58 (1993) 945;  
I.M. Dremin, R.C. Hwa, Phys. Rev. D49 (1994) 5805.
- [17] A. Capella, I.M. Dremin, J.W. Gary, V.A. Nechitailo, J. Tran Thanh Van, Phys. Rev. D61:074009 (2000).

- [18] I.M. Dremin, J.W. Gary, Phys. Lett. B459 (1999) 341.
- [19] I.M. Dremin, C.S. Lam, V.A. Nechitailo, Phys. Rev. D61:074020 (2000).
- [20] I.M. Dremin, Phys. Lett. B313 (1993) 209.